B.Sc Part II (Honors)

Ring of Residue classes.

Q → Prove that the set of residue classes modulo the positive integer m, is a ring w.r.t. addition and multiplication of residue classes (mod m).

Solution:— Let the set of residue classes (mod m) be $I/(m)$.

(i) We know that I/m is an Abelian group w.r.t. addition modulo m

Thus the first postulate is satisfied.

(ii) Also, multiplication mod (m) is associative, since by definition

$([r]\cdot[s])\cdot[t] = [rs]\cdot[t]$

where rs is reduced modulo if necessary $= [rst]$ " (rst) " " "

where $[r], [s], [t] \in I/(m)$.

Similarly $[r]\cdot([s]\cdot[t]) = [rst]$

In fact the associativity of multiplication mod (m) follows from the fact that the factors involved within the brances are positive integers and the product of positive integers is associative.

(iii) The distributive laws w.r.t. addition and multiplication are satisfied on the same ~~ground~~ ground as above, viz, the numbers involved are positive integers and they are distributive w.r.t. addition and multiplication.

(iv) The Unit element is obviously [1].

Hence the set $I/(m)$ is a commutative ring with unity.

Ring of matrices:

Q→ Show that the set M of all 2×2 matrices where a, b, c, d, are real numbers is a ring with respect to addition and multiplication of matrices.

let $A, B \in M$. Then $A+B \in M$ and $AB \in M$ since the sum and product of 2×2 matrices are 2×2 matrices.

Therefore M is closed with respect to addition and multiplication of matrices.

Solution: — let M be the set of square matrices of order 2.
let $A, B, C \in M$.

Then we know from our knowledge of Algebra that

Laws of addition.

(i) $A+B \in M$ (ii) $(A+B)+C = A+(B+C)$ (Associative law)

(iii) There exists an identity matrix (zero matrix) of order 2 i.e. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ denoted by $0 \in M$ such that for each $A \in M$ we have $A+0 = 0+A = A$.

(iv) There exists a negative matrix $-A \in M$ corresponding to each $A \in M$ such that $A+(-A) = (-A)+A = 0$

(v) $A+B = B+A$. Thus we find that the set M of square matrices of order 2 is an Abelian group w.r.t. addition.

Laws of multiplication: (vi) $(AB)C = A(BC)$ (Associative law)

(vii) Distributive law: $A(B+C) = AB+AC$.

Taking all these conditions (i) to (vii), we find that the set of all square matrices of order 2 is a ring.

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